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# ALTERNATING-CURRENT NETS

BY ARTHUR E. KENNELLY

# ALTERNATING-CURRENT NETS

By Arthur E. Kennelly

Communicated 13th March 1929

*Purpose and Scope*

It is the purpose of this paper to show that every network or *net* of conducting elements, in the steady state, operated between a pair of input- and a pair of output-terminals, at a single impressed frequency, including zero frequency or continuous current as the simplest case, is replacable by two pairs of $T$'s and $\Pi$'s, called respectively the *basic* and the *alternative* pair. In the continuous-current case, the $T$ and $\Pi$ of the alternative class have arithmetical significance only, and are physically unrealisable; but in alternating-current cases this distinction may fail. Realisability may attach to either or both classes. In simple alternating-current transformers, neither class fails.

A three-terminal alternating-current net corresponds to a particular family of plane triangles, a property that had previously been apparently limited to continuous-current cases, or to especially simple alternating-current cases.

*Continuous-Current Four-Terminal Nets*

In any net, such as the rectangular net in Fig. 1, with 24 resistance elements, input terminals $ag$, and output terminals $bh$, it is known that the angle $\theta$ of the net between these pairs of terminals is defined by either of the two equations:

$$\tanh \theta = \sqrt{\frac{R_{ag}}{R_{af}}} = \sqrt{\frac{R_{bg}}{R_{bf}}} \qquad \text{numeric } \angle \qquad (1)$$

where $R_{af}$ and $R_{ag}$ are the net resistances measured at terminals $ag$, with the $bh$ terminals respectively opened and shorted, and similarly for the resistances $R_{bf}$ and $R_{bg}$ at the $bh$ terminals.

In the simple example of Fig. 1, the net reduces to the six-element structure in the lower part of the diagram, $AG$, $BH$, for which

$$\tanh^2 \theta = \frac{2428.57}{6000} = \frac{2833.\dot{3}}{7000} = 0.40476 \qquad (2)$$

or

$$\tanh \theta = 0.636209 \qquad (3)$$

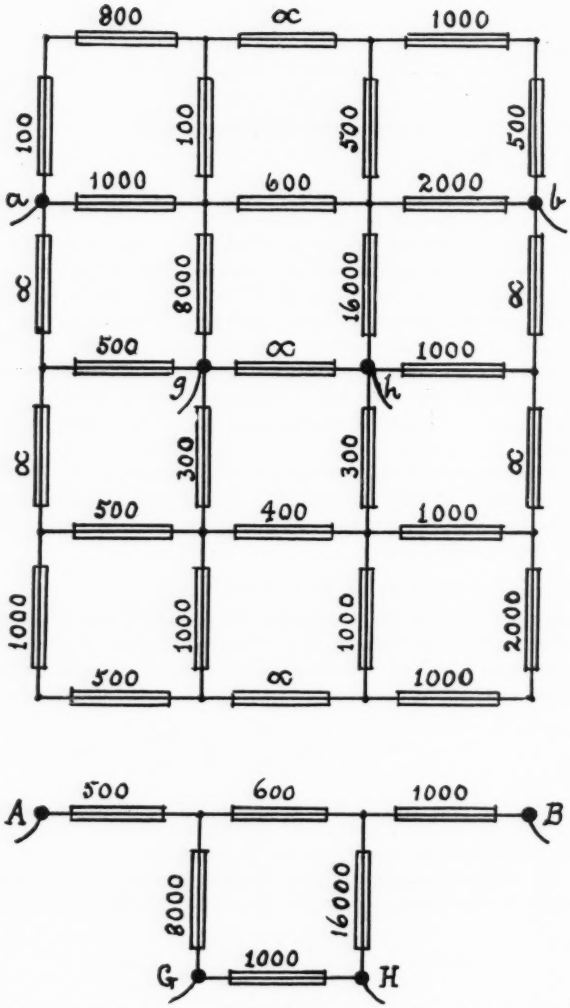



FIGURE 1. Particular Rectangular D. C. Net and its Simplified Counterpart.

The ordinary solution or *basic solution* of (3), by the use of Tables of real hyperbolic functions, is:

$$\theta = 0.751779 \text{ hyperbolic radian, or hyp.} \tag{4}$$

which is the net angle for continuous currents, and these particular pairs of terminals. There is, however, another solution for (3), which may be called the *alternative solution;* namely,

$$\theta = 0.751779 \pm j\pi = 0.751779 \pm j\underline{2} \quad \text{hyps. } \angle \tag{5}$$

That is, we may add or subtract $j\pi$ hyp. radians or $\pi$ circular radians ($\underline{2}$ circular quadrants) to the angle $\theta$, without altering its tangent, since tanh $\theta$ is a periodic function of period $j\pi$. This alternative solution of (3) is here the subject of discussion, especially in relation to alternating-current nets.

Table I presents the principal functions of the two values of $\theta$.

TABLE I

| Basic | Alternative |
|---|---|
| $\theta = 0.751779$ | $\theta = 0.751779 \pm j\underline{2}$ |
| $\sinh \theta = 0.824621$ | $\sinh \theta = -0.824621$ |
| $\cosh \theta = 1.296148$ | $\cosh \theta = -1.296148$ |
| $\tanh \theta = 0.636209$ | $\tanh \theta = 0.636209$ |

It is evident that adding or subtracting $j\underline{2}$ from $\theta$, changes the sign of both the sine and the cosine; but leaves the tangent unaltered.

*Alternative Equivalent Circuits corresponding to the two Values of* $\theta$

It is known that if we define the surge impedances of the net $z_{oa}$, $z_{ob}$, at each end, as the vector geometrical mean of the measured values $R_f$ and $R_g$, we have:

$$z_{oa} = \sqrt{R_{ag} \cdot R_{af}} = \sqrt{2428 \cdot 57 \times 6000} = 3817 \cdot 26 \text{ ohms} \tag{6}$$

$$z_{ob} = \sqrt{R_{bg} \cdot R_{bf}} = \sqrt{2833 \cdot 33 \times 7000} = 4453 \cdot 46 \text{ ohms} \tag{7}$$

also the geomean surge impedance becomes

$$z_{ab} = \sqrt{z_{oa} \cdot z_{ob}} = \sqrt{3817 \cdot 26 \times 4453 \cdot 46} = 4123 \cdot 11 \text{ ohms} \tag{8}$$

or

$$y_{ab} = 1/z_{ab} = 0.242535 \times 10^{-3} \text{ mho.} \tag{9}$$

*Duplicate Equivalent T's*

We proceed to form the duplicate equivalent $T$'s of the net in Fig. 1, in the manner indicated by Figs. 2 and 3. The rule, in all cases, is to place in the staff of the equivalent $T$ an admittance of $y_{ab} \cdot \sinh \theta$ mhos. According to Table I, there are two values for $\sinh \theta$ in this example; namely, 0.824621 and $-$ 0.824621. There are thus two duplicate values for the staff admittance, differing in sign; namely $0.2 \times 10^{-3}$ and $-0.2 \times 10^{-3}$ mho, corresponding to 5000 and $-$ 5000 ohms respectively. These duplicate values appear in Figs. 2A and 2B.

There is of course no arithmetical difficulty in employing at o' o, Fig. 2B, a negative resistance of $-$ 5000 ohms; but physically, the concept is meaningless, in the absence of generative apparatus or electric source in the staff. We are therefore physically prevented from using the alternative solution (3) and the equivalent $T$ pertaining to the same; but since we shall find that the alternative solution is arithmetically valid, we may proceed to complete it, for subsequent extension to alternating-current cases.

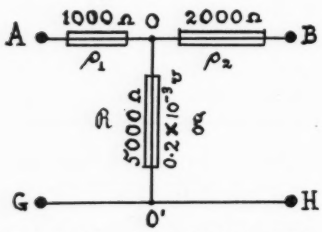



FIGURE 2A. Basic Equivalent T.

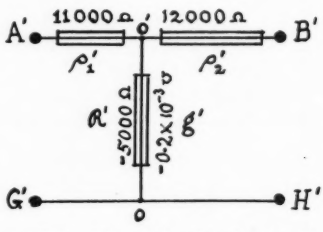



FIGURE 2B. Alternative Equivalent T.

Completing the duplicate $T$'s so as to comply with the measured values $R_{af} = 6000$ and $R_{bf} = 7000$ ohms, we have in Fig. 2A, $\rho'_1 = Ao = 1000$ and $\rho_2 = oB = 2000$ ohms; while in Fig. 2B, $\rho'_1 = A'o = 11000$ and $\rho'_2 = o'B' = 12000$ ohms. It will be observed that

$$\rho_2 - \rho_1 = \rho'_2 - \rho'_1 \qquad \text{ohms} \angle \quad (10)$$

or the arm differences are the same in both $T$'s. It will then be found that the two $T$'s are arithmetically equivalent. That is, either can be used to replace the net of Fig. 1 between terminals $AG$ and $BH$, with any assigned external sources or loads.

*Comparative Behavior of the Basic and Alternative T's*

The effect of changing from the basic to the equivalent $T$ is to reverse the polarity or direction of voltage and current at output terminals, leaving their product, the output power, unchanged in sign. Thus, in Figs. 3A and 3B, an emf. of 1.0 volt is impressed at input terminals and a load of $\sigma = 1000$ ohms is connected to the output terminals. In the basic $T$, the output emf. is 0.217391 volt and the output current 0.217391 milliampere. In the alternative $T$, these values have the negative sign, or the opposite direction. The effect in the load $\sigma$ is such as might be produced by reversing the output terminals.

Again, if we examine the position angles at output and input terminals in the two cases, we have the position angle $\delta_b$ at $BH$, from the known relation:

$$\delta_b = \tanh^{-1}\left(\frac{\sigma}{z_{ob}}\right) \qquad \text{hyps } \angle \quad (11)$$

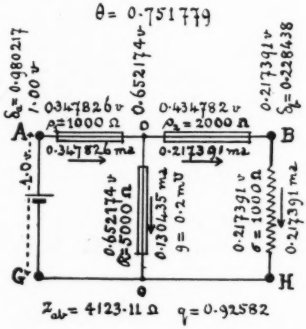



FIGURE 3A. Basic Equivalent T.

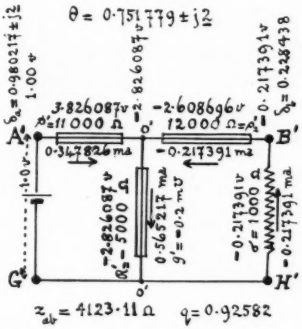



FIGURE 3B. Alternative Equivalent T. Load of 1000 ohms at BH, and 1.0 volt impressed at AG.

This applies to both the basic and alternative $T$; but the position angle $\delta_a$, at $AG$, which is always $\theta + \delta_b$, is different in the two cases. It is 0.980217 in Fig. 3A, and $0.980217 \pm j\underline{2}$ in Fig. 3B. The sine and cosine of the latter angle will both be essentially negative. Consequently the equations for output voltage and current:

$$E_b = \frac{E_a}{q} \cdot \frac{\sinh \delta_b}{\sinh \delta_a} \qquad \text{volts } \angle \quad (12)$$

and

$$I_b = I_a q \frac{\cosh \delta_b}{\cosh \delta_a} \qquad \text{amperes} \angle \quad (13)$$

must give positive continuous-current values with the basic $T$ and negative values with the alternative $T$.

*Duplicate Equivalent* Π*'s.*

If we insert in the architrave of the equivalent Π, according to regular rule, the impedance

$$\rho = z_{ab} \sinh \theta \qquad \text{ohms} \angle \quad (14)$$

there will be two duplicate Π's, using the two values of sinh θ from Table I. These are represented in Figs. 4 and 5. In $AB$, we insert

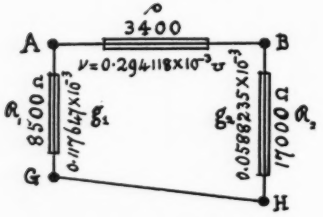



FIGURE 4. Basic Equivalent Π.

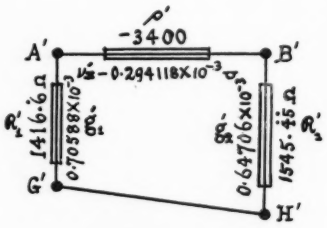



FIGURE 5. Alternative Equivalent Π.

$4123.11 \times 0.824621 = 3400$ ohms; while in $A'B'$, we insert arithmetically the negative of this, or $-3400$ ohms. The pillar leaks must then be selected such that

$$g_1 + \nu = G_{ag} = \frac{1}{R_{ag}} \qquad \text{mhos} \angle \quad (15)$$

and

$$g_2 + \nu = G_{bg} = \frac{1}{R_{bg}} \qquad \text{mhos} \angle \quad (16)$$

Here again, the duplicate equivalent Π's are equally valid arithmetically; but the alternative Π, with its negative resistance architrave is physically meaningless and unrealisable, in the absence of some generating device.

There are thus four three-element equivalent structures which arithmetically simulate the behavior of a given four-terminal net,

of which two only are physically realisable in the continuous-current case. The effect of changing from the basic to the alternative structures, is to reverse the direction of voltage and current at output terminals.

*Alternating-Current Nets*

Every single-frequency alternating-current net, in the steady state, whose elements individually obey alternating-current Ohm's law,* is known to subtend a complex angle $\theta = \theta_1 + j\theta_2$, as defined by formula (1); where the measurable impedances $R_{af}$, $R_{ag}$, $R_{bf}$ and $R_{bg}$, are all complex quantities. As, in the continuous-current case, equation (1) admits of two solutions, one basic and the other alternative, and the latter value of $\theta$ differs from the former by $\pm \pi$ in the imaginary component; i. e. 2 quadrants, 200 grads, or 180 degrees

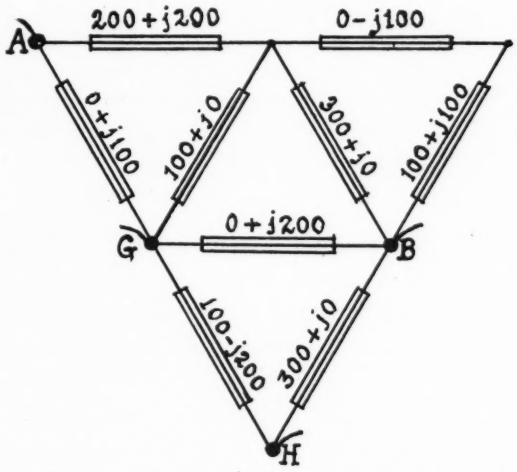



FIGURE 6. Particular Nine-element A. C. Net.

Thus, the particular nine-element net of Fig. 6, gives rise to the relations: $R_{ag} = 84{\cdot}070 \angle 78° 47' 55''$

$$R_{bg} = 134{\cdot}063 \sphericalangle 19° 12' 41'' \text{ ohms } \angle \qquad (17)$$

* Bibliography 1a.

$$R_{af} = 83{\cdot}5417 \angle 79° 13' 56''$$

$$R_{bf} = 133{\cdot}220 \measuredangle 18° 46' 40'' \quad \text{ohms} \angle \tag{18}$$

The ratio of either of these pairs of terminal impedances gives, following (2) and (3),

$$\tanh^2 \theta = 1{\cdot}00632 \measuredangle 0° 26' 1'' \quad \text{numeric} \angle \tag{19}$$

and

$$\tanh \theta = 1{\cdot}00316 \measuredangle 0° 13' 0.5'' \quad \text{numeric} \angle \tag{20}$$

The basic solution of this, by Tables or Charts,* is

$$\theta = 3{\cdot}003 + j2{\cdot}007 = 3{\cdot}003 + j\underline{1{\cdot}279} \quad \text{hyps} \angle \tag{21}$$

and the alternative solution is

$$\theta = 3{\cdot}003 - j1{\cdot}135 = 3{\cdot}003 - j\underline{0{\cdot}721} \quad \text{hyps} \angle \tag{22}$$

Table II gives the principal functions of $\theta$ for the two cases.

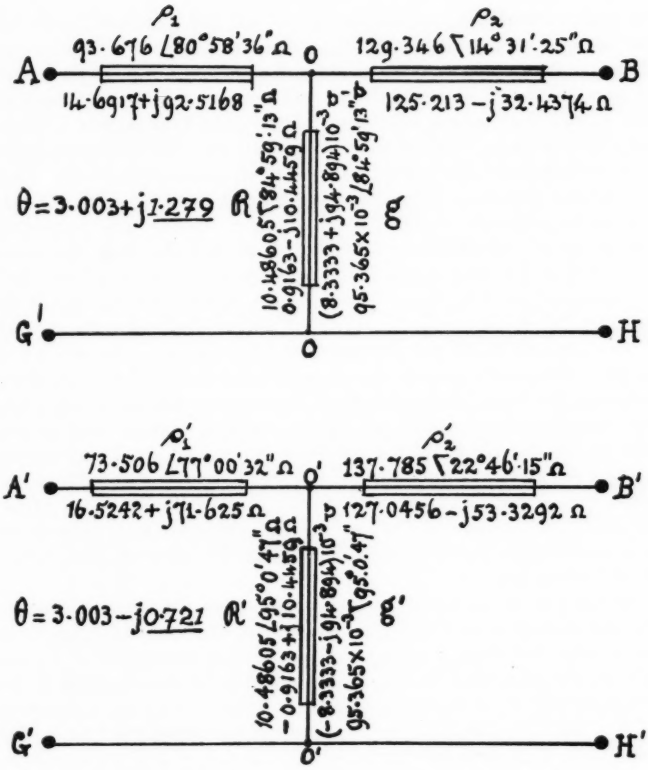
TABLE II

| | Basic | Alternative |
|---|---|---|
| $\theta$ | $3.003 + j\,\underline{1.279}$ | $3.003 - j\,\underline{0.721}$ |
| $\sinh \theta$ | $10.0924 \angle 114° 59' 51''$ | $-10.0924 \angle 114° 59' 51''$ or $10.0924 \measuredangle 65° 0' 9''$ |
| $\cosh \theta$ | $10.0607 \angle 115° 12' 52''$ | $-10.0607 \angle 115° 12' 52''$ or $10.0607 \measuredangle 64° 47' 8''$ |
| $\tanh \theta$ | $1.00316 \measuredangle 0° 13' 1''$ | $1.00316 \measuredangle 0° 13' 1''$ |

If we take the basic form, the staff admittance for the equivalent $T$ is $y_{ab} \sinh \theta = 9.44921 \times 10^{-3} \measuredangle 30° 00' 38'' \times 10.0924 \angle 114° 59' 51'' = 95.3648 \times 10^{-3} \angle 84° 59' 13''$ mho, or $10.48605 \measuredangle 84° 59' 13''$ ohm, as shown at $g$ and $\mathscr{R}$ in Fig. 7. The two arm impedances $\rho_1$ and $\rho_2$ are then found by subtracting $\mathscr{R}$ from $R_{af}$ and $R_{bf}$ respectively.

If we take the alternative form, as in Fig. 8, the staff admittance $g'$ has the same size, but its slope differs from that of $g$ Fig. 7, by 180°. In this case the basic $T$ is realisable: while the alternative $T$ is physically unrealisable, its staff impedance having a slope exceeding 90°, or a negative resistance component. Arithmetically, however, both $T$'s are realisable. Both would yield the same terminal condi-

* Bibliography 3 and 4.




FIGURES 7 and 8. Basic and Alternative Ts of Net in Fig. 6.

tions as to voltage, current and power, when connected up to output and input in the same manner as the actual net.

Proceeding to form the equivalent Π of the net, the architrave receives either of the duplicate impedances $z_{ab} \cdot \sinh \theta = 105.829 \angle 30° 00' 38'' \times \pm 10.0924 \angle 114° 59' 51'', = 1068.073 \angle 145° 00' 29''$ or 1068.073 ∠ 34° 59′ 31″ ohms, as in Figs. 9 and 10. Here the basic Π happens to be physically unrealisable; while the alternative Π is realisable. The net of Fig. 6 might thus be replaced in any

external or enveloping net, using the $AG$ and $BH$ terminals, by either the basic $T$ or the alternative $\Pi$. The change from a basic to an alternative form of structure has the effect of reversing the direction, or sign, of the delivered voltage and current. The actual net of Fig. 6 would of course offer no ambiguity in this respect. When supplied with say 1 volt of alternating emf. and with standard phase at input terminals $AG$, it would deliver to an ammeter short circuiting the output terminals $BH$, a current definite both as to magnitude and phase; namely 0.936265 milliampere, leading the impressed voltage at $AG$ by a phase of 34° 59′ 31″, as indicated over the architrave in Fig. 9B, and not with the opposite phase of −145° 00′ 29″, as indicated in Fig. 9A. Therefore the alternative $\Pi$ would be justified; while the basic $\Pi$ would be disqualified. In the absence of any tests to reveal the difference of phase between the input and output voltages or currents, there would be no means of distinguishing between the basic and equivalent structures. Either would yield the same apparent values of output voltage, current and power. The ambiguity as to the phase difference between input and output voltage or current is inherent in the system, until eradicated by some kind of comparative phase test. In the absence of such tests, either the basic or alternative $T$'s and $\Pi$'s may be used to represent the net, according to their realisability, or capability of being constructed in the laboratory.

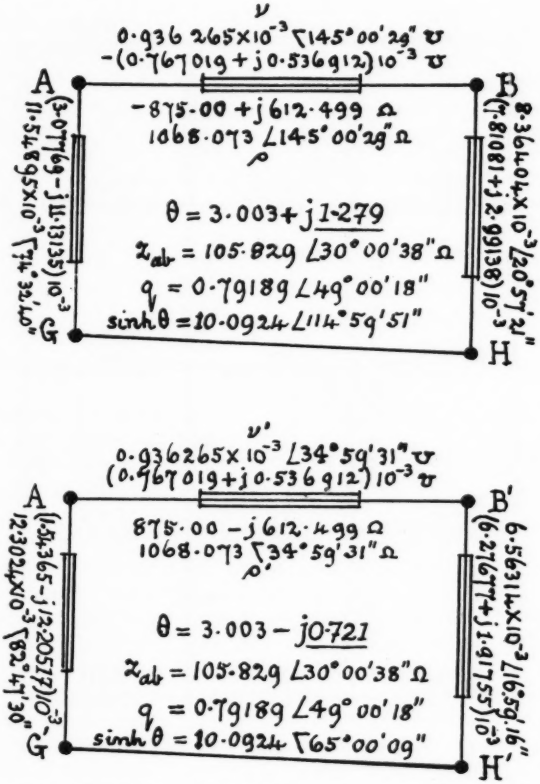
In the case of an induction coil or air-cored transformer entering an alternating-current net, with primary input terminals and secondary output terminals, it is known that the same ambiguity of $T$'s and $\Pi$'s exists,* according to the relative direction of the windings in the coil with respect to the output terminals. In such a case, the ambiguity is not only inherent, as in other alternating-current cases; but is actually necessary, in order to explain the behavior of the apparatus in nets.

*Three-Terminal Nets*

If the second output terminal $H$ is connected to the same point of the net as the second input terminal $G$, the four-terminal net becomes reduced to a three-terminal net as in Fig. 10, where output terminal $H$ of Fig. 1 has been moved to coincide with input terminal $G$. In such a case, it is† known that for any continuous-current net there-

* Bibliography 6.

† Bibliography 7.




FIGURES 9A and 9B. Basic and Alternative IIs of Net in Fig. 6.

fore devoid of reactance, the net resolves itself into a simple delta connection of three resistances *ABC*. In Fig. 11, the delta *ABC* results from the three-terminal net of Fig. 10. Keeping the testing leads *a*, *b* and *gh* fixed, and rotating the net clockwise into its three aspects, as shown in the Figure, the three geomean surge resistances are each and all equal to 3470.30 ohms, the three *q* inequality factors have unity as their product; while the three angles of the net, in its three successive aspects are such that the sum of their sinhs is 8.18249.

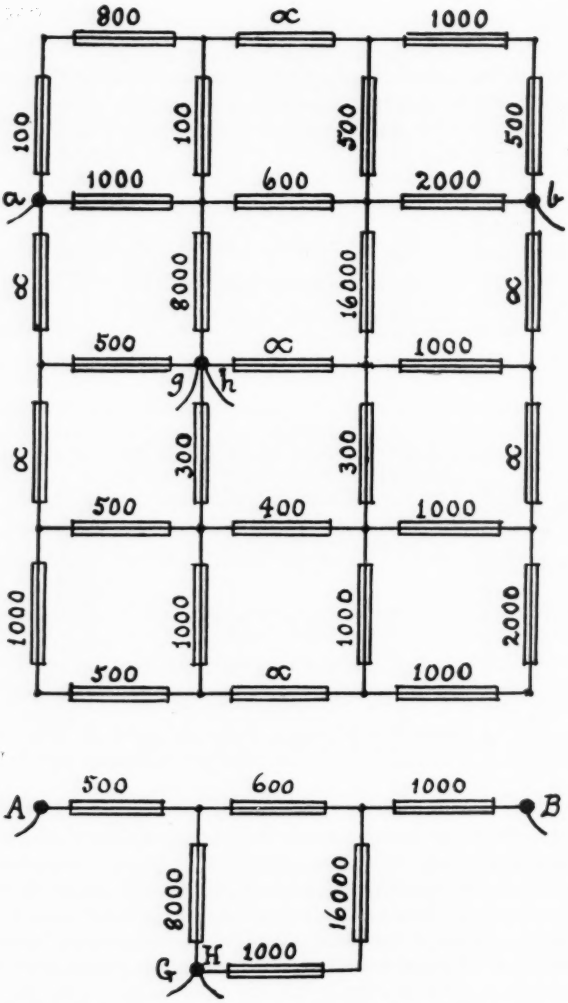



FIGURE 10. Particular Rectangular D. C. Net and its Simplified Counterpart.

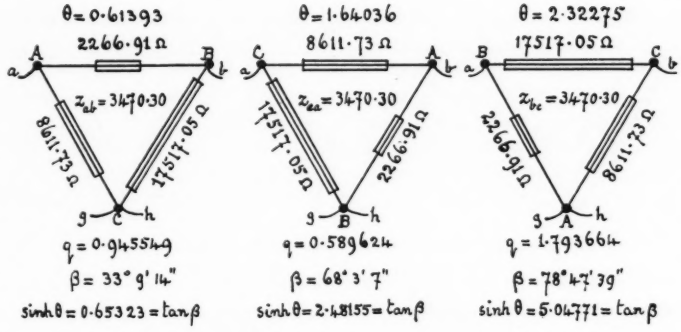



FIGURE 11. Three-terminal Net of Fig. 10 and its Three Aspects.

which is likewise their product. This means that the three corresponding gudermannian angles* of the net add up to 180°, or correspond to the three interior angles of a certain family of plane triangles. Although this gudermannian property has already been recognized in relation to continuous-current nets, it had to be relinquished for the general alternating-current net; because the three aspect angles θ of an alternating-current net being, in general, complex, would involve complex gudermannian circular angles β. Complex gudermannians have only recently been investigated;† so that having given any complex hyperbolic angle θ, its complex gudermannian circular angle β may be forthwith ascertained. It now appears that in the general case of an alternating-current three-terminal net, the three complex gudermannians are such that the three real components add up to 180°, or define a plane triangle; while the three imaginary components add up to zero, or mutually cancel in the additive process.

*A. C. Three-Terminal Nets*

A particular case of an alternating-current three-terminal net is represented in its three aspects by Fig. 12. The numerical details appear in Table III. It will be seen that there are six pairs of measured terminal impedances, or twelve values in all. These give rise to six different end surge impedances, from which the three geomean surge impedances are identical. The three inequality

* Bibliography 7.

† Bibliography 8.

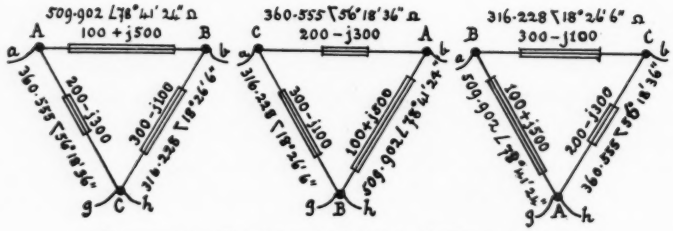



FIGURE 12. Three-terminal Net in its Three Aspects.

TABLE III

DATA FOR THE THREE-TERMINAL ALTERNATING-CURRENT NET A B C OF FIG. 12

| | Aspect $BC$ | Aspect $AB$ | Aspect $CA$ |
|---|---|---|---|
| $R_{ag}$ | $285.044 \angle 15° 15' 18''$ | $509.902 \diagdown 11° 18' 36''$ | $178.065 \diagdown 36° 5' 6''$ |
| $R_{af}$ | $536.757 \angle 30° 34' 5''$ | $335.309 \diagdown 20° 46' 20''$ | $187.444 \angle 5° 47' 34''$ |
| $R_{bg}$ | $178.065 \diagdown 36° 5' 6''$ | $285.044 \angle 15° 15' 18''$ | $509.902 \diagdown 11° 18' 36''$ |
| $R_{bf}$ | $335.309 \diagdown 20° 46' 20''$ | $187.444 \angle 5° 47' 34''$ | $536.757 \angle 30° 34' 5''$ |
| $z_{0a}$ | $391.151 \angle 22° 54' 41''$ | $413.491 \diagdown 16° 2' 28''$ | $182.694 \diagdown 15° 8' 46''$ |
| $z_{0b}$ | $244.350 \diagdown 28° 25' 43''$ | $231.149 \angle 10° 31' 26''$ | $523.157 \angle 9° 37' 44''$ |
| $z_{ab}$ | $309.157 \diagdown 2° 45' 31''$ | $309.157 \diagdown 2° 45' 31''$ | $309.157 \diagdown 2° 45' 31''$ |
| $q$ | $1.26521 \angle 25° 40' 12''$ | $1.33748 \diagdown 13° 16' 57''$ | $0.590945 \diagdown 12° 23' 15''$ |
| $1/q$ | $0.790376 \diagdown 25° 40' 12''$ | $0.747674 \angle 13° 16' 57''$ | $1.692207 \angle 12° 23' 15''$ |
| $\sinh\theta$ | $1.64933 \angle 81° 26' 55''$ | $1.022872 \diagdown 15° 40' 35''$ | $1.166254 \diagdown 53° 33' 5''$ |
| | $0.245249 + j\,1.630996$ | $0.984824 - j\,0.276384$ | $0.692873 - j\,0.938122$ |
| $\theta$ | $1.09336 + j\,1.38457$ | $0.884274 - j\,0.196290$ | $0.843072 - j\,0.749550$ |
| | $1.09336 + j\,0.881416$ | $0.884274 - j\,0.124962$ | $0.843072 - j\,0.477178$ |
| | $1.09336 + j\,70° 19' 39''$ | $0.884274 - j\,11° 14' 48''$ | $0.843072 - j\,42° 56' 46''$ |
| $gd\,\theta = \beta$ | $1.431919 + j\,0.680900$ | $0.797141 - j\,0.138505$ | $0.912532 - j\,0.542396$ |
| | $0.911588 + j\,0.680900$ | $0.507475 - j\,0.138505$ | $0.580937 - j\,0.542396$ |
| | $82° 2' 35'' + j\,0.680900$ | $45° 40' 22'' - j\,0.138505$ | $52° 17' 3'' - j\,0.542396$ |

$$\Sigma\beta = \pi = 3.141592 = 2.000000 = 180° 0' 0''$$

$$\Sigma \sinh\theta = \Pi \sinh\theta = 1.922946 + j\,0.416490$$

factors $q$ have $1.0 \angle 0°$ for their product, the sum of their slopes being 0°. The values of the three net angles $\theta$, are given with their imaginary components in circular radians, circular quadrants and sexagesimal degrees. The underscored values are quadrantal. Similarly, the gudermannians $\beta$ of the net are given with their real components in three forms. The sum of these three gudermannians is evidently $\pi$ radians, 2.0 quadrants or 180 degrees. The plane triangle in Fig. 13 represents at $ABC$, the three real components of

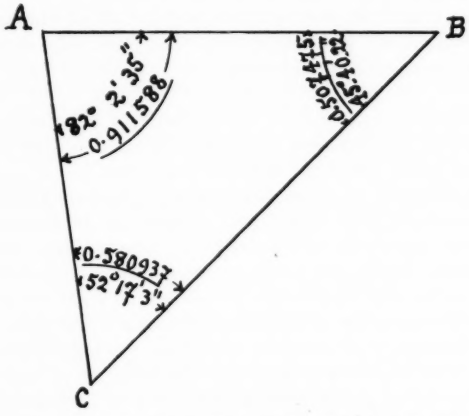



FIGURE 13. Plane Triangle Corresponding to the Three-terminal Alternating-current Net of Fig. 12.

the gudermannians $\beta$, whose imaginary components add up to* zero.

The results of this particular three-terminal *a. c.* net are of general application.

*Summary of Results*

1. Any net of more than three elements, having two input and two output terminals, and carrying steady currents of a single frequency (including zero frequency, or continuous currents as a limiting case)

* The author is indebted for assistance in computing the case of Figs. 12 and 13, with Table III, to a group of five Chinese students, who undertook the task in subgroups, so organized as to furnish mutual checks upon the accuracy of the arithmetical work. Their names are Yun Chu, Pao G. Shen, Pinlu Shen, Shu Z. Shen and Tsen C. Tsao.

may be replaced by any one of four three-element artificial structures; viz, two $T$'s and two $\Pi$'s, that may be separated into two classes,—basic and alternative.

2. In the continuous-current case, the basic $T$ and $\Pi$ are always physically realisable, and the alternative $T$ and $\Pi$ physically unrealisable.

3. The change from a basic to an alternative structure, or *vice versa*, reverses the direction or sign of the output voltage and current.

4. In any alternating-current net, any or all of the four arithmetically equivalent structures may be physically unrealisable. A physically realisable *a.c.* structure may be either basic or alternative.

5. The change from a basic to an alternative *a. c.* structure involves a change of 180° in the phases of output voltage and current, without affecting the phase difference between these two.

6. The ambiguity of phase between input and output voltage or current associated with basic and alternative structures, can be cleared up by some test which compares these phases. Such a test may disqualify either the basic or alternative structures.

7. Both basic and alternative structures may be needed for the adequate representation of nets containing air-cored induction coils.

8. The ambiguity of artificial structures pertaining to a net, is associated with the ambiguity of $j\pi$ radians, or $j\underline{2}$ quadrants, in assigning the value of a hyperbolic angle whose tangent is specified.

9. Three-terminal nets, whether carrying continuous or alternating currents, are hypothetically capable of being connected in three different aspects to input and output. The three corresponding net angles $\theta$, have complex gudermannian angles $\beta$, whose imaginary components add up to zero, and whose real components add up to 180° or define a family of plane triangles.

10. In any symmetrical four-terminal net, the change from basic to alternative structure involves the substitution of $z_0 \coth (\theta/2)$ for $z_0 \tanh (\theta/2)$ in the T-arm impedances, and of $y_0 \coth (\theta/2)$ for $y_0 \tanh (\theta/2)$ in the $\Pi$-pillar admittances.

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## LIST OF SYMBOLS EMPLOYED

$\beta$—Real gudermannian angle corresponding to real net hyp. angle (cir. radians).

$\beta = \beta_1 + j\beta_2$—Complex gudermannian angle corresponding to complex hyp. angle of an alternating-current net (radians $\angle$).

$\delta_a$, $\delta_b$—Position angles at input and output terminals of a net, under an assigned output load (hyps $\angle$).

$E_a$, $E_b$—Voltages at input and output terminals of a net (*rms.* volts $\angle$).

$G_{ag}$, $G_{bg}$—Admittances at input and output terminals, with opposite-end terminals shorted (mhos $\angle$).

$g_1$, $g_2$—Pillar admittances of an equivalent $\Pi$ (mhos $\angle$).

$\theta = \theta_1 + j\theta_2$—Hyperbolic angle of a net (hyps $\angle$).

$I_a$, $I_b$—Input and output currents (amperes $\angle$).

$j = \sqrt{-1}$

$\nu$—Architrave admittance of an equivalent $\Pi$ (mhos $\angle$).

$\pi = 3.14159 \ldots$

$q$—Inequality coefficient of a net (numeric $\angle$).

$R_{af}$, $R_{ag}$—Impedances measured at input terminals when the output terminals are successively open and shorted (ohms $\angle$)

$R_{bf}$, $R_{bg}$—Impedances measured at output terminals when the input terminals are successively open and shorted (ohms $\angle$).

$\mathscr{R}$—Staff impedance of an equivalent $T$ (ohms $\angle$).

$\rho_1$, $\rho_2$—Arm impedances of an equivalent $T$ (ohms $\angle$).

$\rho$—Architrave impedance of an equivalent $\Pi$ (ohms $\angle$).

$\Sigma$—Sign of summation.

$\sigma$—Impedance of a load connected to output terminals (ohms $\angle$).

$z_{oa}$, $z_{ob}$—Surge impedances of a net as found at input and output terminals respectively (ohms $\angle$).

$z_{ab}$—Geomean surge impedance of a net (ohms $\angle$).

*gd* $\theta$—Gudermannian angle of the hyperbolic angle $\theta$ (radians, degrees or grads).

$\Pi$—Sign of a product of multiplication, or sign of the equivalent *pi* of a net.

$T$—Sign of equivalent *tee* of a net.

*a. c.* contraction for *alternating current.*